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Condition for the formation of an Ideal Ramanujan Number (IRN) having two wings $a^3 \pm b^3 = c^3 - d^3$ where $(a \pm b) > 1$& $c - d = 1$

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ABSTRACT

This is further little development of the theory by which mystery of Ramanujan number was penetrated and published in this journal IJSER vide April edition 2014. It includes the condition to have a Ramanujan relation of two wings where one wing is the difference of two cubes of two consecutive positive integers and for other wing the algebraic sum of two elements is more than one. Ultimately it may follow equating of infinitely many wings with the difference of two cubes of two consecutive positive integers. With the development of this theory it has been felt necessary to redefine the earlier definition of IRN and RN.

Keywords

Ideal Ramanujan number & Ramanujan number , Rootless factor & rootless zone, Positive & negative wings.

Introduction:

An ideal Ramanujan number (IRN) can be redefined as $N = 1.P_1.P_2.P_3$…….n prime factors including one or $N = 1.(P_1.P_2.P_3…..)^2.Q_1.Q_2.Q_3$…….. n prime factors including one where P_i & Q_i all are in the form of $(6\lambda + 1)$ and where considering all P_i^2 as single prime factor if x be the product of r primes & y is the product of rest (n – r) primes (r < n), the factor $\mu = \sqrt{\{(4x - y^2)/3\}}$ must be an integer at least in two cases i.e. IRN or $N = \{1/2(y_1 + \mu_1)\}^3 + \{1/2(y_1 - \mu_1)\}^3 = \{1/2(y_2 + \mu_2)\}^3 + \{1/2(y_2 - \mu_2)\}^3 = ……….$

Because, we have Ramanujan factor $(R_f) = 12x - 3y^2$ which must be a square integer at least in two cases of the form $(3\mu)^2$ so as to produce Ramanujan relation like $a^3 + b^3 = c^3 + d^3 = ……$ where (a, b) or (c, d) = $½(y \pm \mu)$ & $\mu = \sqrt{\{(4x - y^2)/3\}}$

Accordingly Ramanujan number (RN) can be redefined as $N = (P_1.P_2.P_3……)^3$.(Product of factors, IRN in nature) where P_i are in the form of $(6\lambda + 1)$ or 2 or 3.

Obviously, for an IRN having n prime factors the maximum possible wings are $½(2^n - 2) + 1 = 2^{n-1}$ Because, considering all P_i^2 as single prime factor we can divide the n factors into two groups in $½(2^n - 2)$ different ways where each group can produce a distinct wing and a common wing like $(c + 1)^3 + (-c)^3$ against the factor $\mu = \sqrt{\{(4N - 1^2)/3\}}$. By equal distribution of P_i^2 as P_i & P_i to two groups we cannot have integer value of μ. Now the question is whether there exists any IRN_n of maximum number of wings 2^{n-1} for n > 2 or not?
Now, IRN_2 which has two prime factors x & y (x < y) i.e. N = xy can produce not more than two wings.
If x is of 1st kind form i.e. in the form of $4\lambda - 1$, y may be 1st kind or may be 2nd kind but if x is of 2nd kind i.e. in the form of $4\lambda + 1$, y is bound to be 1st kind. Obviously, $y > (x/2)^2$
Now, if $\mu_1 = \sqrt{\{(4y - x^2)/3\}}$ & $\mu_2 = \sqrt{\{(4xy - 1)/3\}}$ both are integers N will produce an ideal Ramanujan relation of two wings like
$N = \{½(x + \mu_1)\}^3 + \{½(x - \mu_1)\}^3 = \{½(1 + \mu_2)\}^3 + \{½(1 - \mu_2)\}^3$
Obviously, RH wing is negative wing of two consecutive numbers and the LH wing will be positive or negative wing according as y lies in between $(x/2)^2$ & x^2 or greater than x^2. Here, both the sides are combination of odd & even elements.
If y = x + k then $4(x + k) - x^2 = 3\mu_1^2$ & $4x(x + k) - 1 = 3\mu_2^2$. Eliminating k we have $x^3 - 1 = 3(\mu_2^2 - x\mu_1^2)$ which clearly indicates that x is such a prime no. that $x^3 - 1$ is divisible by 3 i.e. x is in the form of $3\lambda_1 + 1$
Now, $4xy = 3\mu^2 + 1$. If y is in the form of $3\lambda_2 + 2$ then $4xy = 3\lambda_3 + 2$ form & there is mismatching of nature between LHS & RHS. If y is in the form of $3\lambda_2 + 1$ then both sides are in the form of $3\lambda_2 + 1$
Hence, x & y both are in the form of $3\lambda + 1$ i.e. $6\lambda + 1 \Rightarrow$The number IRN_2 (= xy) is also in the form of $6\lambda + 1$.
In general all the prime factors of IRN are in the form of $6\lambda + 1$ where there exists at least one factor in the form of $4\lambda - 1$.
The greatest factor of IRN_2 or in each group of IRN_n can be said as rootless factor as it fails to be divided into two integers a & b so that algebraic sum of $a^3 + b^3 = N$. For other two factors i.e. 1 & x there is a possibility to have a + b = 1 or a + b = x & $a^3 + b^3 = N$
$\Rightarrow$ for IRN_n number of rootless factors is $2^{n-1} - 1$
Regarding positive or negative wings we can further restrict the domain of y.
Obviously, $½[x - \sqrt{\{(4y - x^2)/3\}}] \le -1$ or, $y \ge x^2 + 3x + 3$ to have both negative wings.
Again from $½[x - \sqrt{\{(4y - x^2)/3\}}] \ge 1$ we have $y \le x^2 - 3x + 3$ for mixed wings of positive & negative.
Finally we can conclude with respect to x that:
For $0 < y < (x/2)^2$ no real roots exist & can be said as imaginary zone
For $(x/2)^2 < y \le x^2 - 3x + 3$ there exists positive roots & can be said as positive zone.
For $x^2 - 3x + 3 < y \le x^2 + 3x + 3$ there exists no roots & can be said as rootless zone.

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For $y \geq x^2 + 3x + 3$ negative-positive mixed roots are available & can be said as negative zone that produces both negative wings. With respect to x, y exists finitely for mixed wings and infinitely for both sided negative wings. Y may exist as a product of several primes also.
The above analysis is true for any two grouped division of IRN_n

1. Condition to have an IRN of two negative wings $a^3 - b^3 = c^3 - d^3$ with respect to x = a – b (x > 1) where c – d = 1.

We have $a = b + x \Rightarrow a^2 + ab + b^2 = (b + x)^2 + b^2 + b(b + x) = y$ (say)
Or, $3b^2 + 3bx + x^2 = x^2 + 3x + 3 + k$ or, $3b^2 + 3bx - (3x + 3 + k) = 0$.
$\Rightarrow b = 1/6.[-3x \pm \sqrt{\{9x^2 + 12(3x + 3 + k)\}}]$
Obviously, $9x^2 + 36x + 12(3 + k)$ is a square integer.
$\Rightarrow 9(x + 2)^2 + 12k$ is a square integer. Where obviously $k = 3k_1$ form.
$\Rightarrow 9\{(x + 2)^2 + 4k_1\}$ is a square integer
$\Rightarrow (x + 2)^2 + 4k_1$ is a square integer $= l^2$ (say)
$\Rightarrow ¼.\{l^2 - (x + 2)^2\} = k_1$ where l is odd & > (x + 2)
$\Rightarrow ¼\{(2n + x + 2)^2 - (x + 2)^2\} = k_1$ where n = 1,2,3,4,……..
$\Rightarrow n(n + x + 2) = k_1$ & $k = 3n(n + x + 2)$
Hence, $3n(n + x + 2) + (x^2 + 3x + 3)$ must be a prime number or product of prime nos. all are in the form of 6λ + 1 and is expressible in the form of $a^3 - b^3$ where a – b = x.
$\Rightarrow$ if $1/3.[4x\{3n(n + x + 2) + (x^2 + 3x + 3)\} - 1]$ is a square integer it will produce another wing $c^3 - d^3$ where c – d = 1.

2. Condition to have an IRN of two mixed wings $a^3 + b^3 = c^3 - d^3$ with respect to x = a + b (x > 1) where c – d = 1.

Here, a + b = x & $a^2 - ab + b^2 = (x - b)^2 - b(x - b) + b^2 = x^2 - 3x + 3 - k$ (say) where $k = (x^2 - 3x + 3)$
By similar approach we get $(x - 2)^2 - 4k_1$ must be a square integer where $k = 3k_1$ and finally we get the condition that:
$1/3.[4x\{(x^2 - 3x + 3) - 3n(x - n - 2)\} - 1]$ must be a square integer where n = 0,1,2,3,……. & n < (x – 1)/2

Example: suppose x = 7 & for mixed wings of IRN we have
$f(n) = 1/3.[4.7\{31 - 3n(5 - n)\} - 1]$ i.e. $f(n) = (28n^2 - 140n + 289)$ is to be a square integer for n = 0, 1, 2
Now $f(0) = 289 = 17^2$ & hence it produces a wing $\{1/2(1 + 17)\}^3 + \{1/2.(1 - 17)\}^3$ i.e. $9^3 - 8^3$ along with the positive wing $[1/2.\{7 + \sqrt{(4.31 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.31 - 7^2)/3}\}]^3$ i.e. $6^3 + 1^3 \Rightarrow 6^3 + 1^3 = 9^3 - 8^3$.
Again, $f(2) = 11^2$ & it produces a wing $\{1/2(1 + 11)\}^3 + \{1/2.(1 - 11)\}^3$ i.e. $6^3 - 5^3$ along with the positive wing $[1/2.\{7 + \sqrt{(4.13 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.13 - 7^2)/3}\}]^3$ i.e. $4^3 + 3^3 \Rightarrow 4^3 + 3^3 = 6^3 - 5^3$.
Further generation of mixed wings with respect to 7 is not possible.

For both sided negative wings of IRN, we have $f(n) = 28n^2 + 252n + 681$ where $f(1) = 31^2$ & it produces a negative wing $1/2(1 + 31)\}^3 + \{1/2.(1 - 31)\}^3$ i.e. $16^3 - 15^3$ along with the negative wing $[1/2.\{7 + \sqrt{(4.103 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.103 - 7^2)/3}\}]^3$ i.e. $9^3 - 2^3$ $\Rightarrow 16^3 - 15^3 = 9^3 - 2^3$.
Again $f(8) = 67^2$ & it produces a negative wing $1/2(1 + 67)\}^3 + \{1/2.(1 - 67\}^3$ i.e. $34^3 - 33^3$ along with the negative wing $[1/2.\{7 + \sqrt{(4.13.37 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.13.37 - 7^2)/3}\}]^3$ i.e. $16^3 - 9^3 \Rightarrow 16^3 - 9^3 = 34^3 - 33^3 = 7.(13.37)$

Again $f(16) = 109^2$ & it produces a negative wing $\{1/2(1 + 109)\}^3 + \{1/2.(1 - 109\}^3$ i.e. $55^3 - 54^3$ along with the negative wing $[1/2.\{7 + \sqrt{(4.19.67 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.19.67 - 7^2)/3}\}]^3$ i.e. $24^3 - 17^3 \Rightarrow 24^3 - 17^3 = 55^3 - 54^3$.

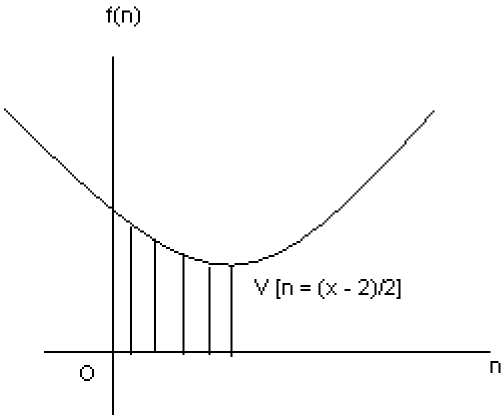
Again $f(41) = 241^2$ & it produces a negative wing $1/2(1 + 241)\}^3 + \{1/2.(1 - 241\}^3$ i.e. $121^3 - 120^3$ along with the negative wing $[1/2.\{7 + \sqrt{(4.7^2.127 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.\ 7^2.127 - 7^2)/3}\}]^3$ i.e. $49^3 - 42^3 \Rightarrow 7^3(7^3 - 6^3) = 121^3 - 120^3 = 7^3.127$

Again $f(196) = 1061^2$ & it produces a negative wing $1/2(1 + 1061)\}^3 + \{1/2.(1 - 1061\}^3$ i.e. $531^3 - 530^3$ along with the negative wing $[1/2.\{7 + \sqrt{(4.103.1171 - 7^2)/3}\}]^3 + [1/2.\{7 - \sqrt{(4.\ 103.1171 - 7^2)/3}\}]^3$ i.e. $204^3 - 197^3 \Rightarrow 204^3 - 197^3 = 531^3 - 530^3$ & so on.

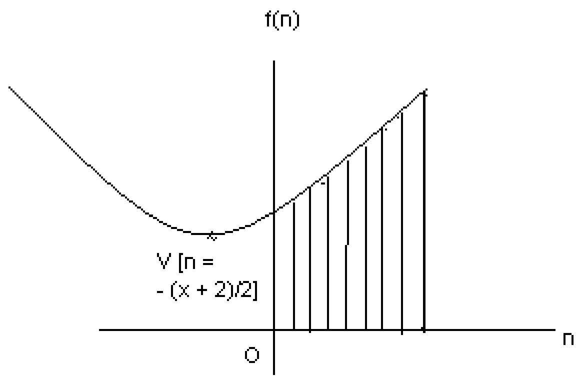
Note: All the prime numbers in the form of 6λ + 1 only can participate as a prime factor to form any IRN or RN and for f(n) of square integer we will get intermittently RN of two wings in the form of x^3.(product of such primes). This is because of the fact that $a^3 - b^3$ where a – b = x attracts all the wings like $x^3\{(p + 1)^3 - p^3\}$ where a – b = x
f(n) clearly indicates that there exits infinitely many primes of the form (6λ + 1) as f(n) is an increasing function with all prime factors in the form of (6λ + 1)
Let us take another example with respect to x = 13
For mixed wings of IRN we have $f(n) = (52n^2 - 572n + 2305)$ which is to be a square integer for n = 0, 1, 2, …., 5
We have only square integer for $f(2) = 37^2$ & it produces a negative wing $19^3 - 18^3$ and a positive wing $10^3 + 3^3 \Rightarrow 10^3 + 3^3 = 19^3 - 18^3 = 13.79$
For both sided negative wings $f(n) = 52n^2 + 780n + 3657$
Where $f(1) = 67^2$ that produces $34^3 - 33^3 = 15^3 - 2^3 = 13.(7.37)$
$f(8) = 115^2$ that produces $58^3 - 57^3 = 22^3 - 9^3 = 13.(7.109)$
$f(19) = 193^2$ that produces $97^3 - 96^3 = 33^3 - 20^3 = 13.(7.307)$
$f(164) = 1237^2$ that produces $619^3 - 618^3 = 178^3 - 165^3 = 13.(43.2063)$ & so on.

With respect to x = 19 & for mixed wings $f(n) = 76n^2 - 1292n + 7777$ & only f(1) is found to be a sq. integer i.e. 81^2 for n < (19 – 2)/2
Hence, it produces a mixed relation $41^3 - 40^3 = 19.(7.37) = 17^3 + 2^3$ where with respect to 7 it fails to produce another wing.
For both sided negative wings, $f(n) = 76n^2 + 1596n + 10665$ where $f(2) = 119^2$, $f(10) = 185^2$, $f(185) = 1705^2$, ……..
Accordingly it produces relations $60^3 - 59^3 = 22^3 - 3^3 = 19(13.43)$; $93^3 - 92^3 = 30^3 - 11^3 = 19(7.193)$; $853^3 - 852^3 = 205^3 - 186^3 = 19(7.13^2.97)$ …………

3. Graphical representation of f(n) with respect to x.



Finite nos. of mixed wings w.r.to x



Both sided negative wings w.r. to x

Conclusion:

With the help of this method first we get a two-wings' relation with respect to x i.e. $N = xy = a^3 \pm b^3 = c^3 - d^3$ where $c - d = 1$ & $a \pm b > 1$ & $x < y$
Now, if y is found to be again product of several primes then there is a possibility to have other wings that equate with $c^3 - d^3$. Those wings can be obtained by two-grouped division method i.e. $\sqrt{[\{(4y_i - x_i^2)/3]} \in I^2$.
Example: with respect to 7 we have received $13.(7.37) = 15^3 - 2^3 = 34^3 - 33^3$. Now, from $\sqrt{[\{(4.13.37 - 7^2)/3]} \in 25^2$, we get another wing that equates with $34^3 - 33^3$ i.e. $16^3 - 9^3 = 15^3 - 2^3 = 34^3 - 33^3$.
Hence, all the relations of multi wings that equate with the difference of two cubes of consecutive elements are obtained by this 'f(n)-square' theory.

References

Books

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[2] In April edition, Vol-5 & issue 4 of IJSER, Author DEBAJIT DAS,

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